On the Synthesis of Equivalent-Circuit Models for Multiports Characterized by Frequency-Dependent Parameters

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***Abstract*—The synthesis of lumped-element equivalent circuits for time-domain analysis of problems with frequency-dependent parameters is of great interest in microwave theory. This paper presents a systematic approach to generate minimal-order realizations for passive microwave circuits characterized by either admittance, impedance, or scattering-parameter data. In addition, a very efficient method to ensure inherent system properties such as stability and passivity is described. The proposed method is suitable for simulating frequency-response-based linear subnetworks in a general circuit environment consisting of lumped/distributed elements and nonlinear devices. Modeling examples for a two-, four-, and ten-port system with nonlinear and linear terminations, respectively, are given.**

***Index Terms*—Curve fitting, equivalent circuits, frequency-domain synthesis, modeling, multiport circuits, passive circuits, reduced-order systems.**

I. Introduction

EFFICIENT transient simulations of components characterized by measured or tabulated frequency-dependent data have been widely discussed in microwave literature. Especially the modeling of multi-input multi-output (MIMO) systems is still considered to be a challenge. Because of the high density and complexity of modern microwave devices, the use of reduced-order macromodels is imperative for efficient circuit simulation for the purposes of signal-integrity assessment and electrical-functionality verification [1]–[8]. The major difficulty usually encountered when linking the systems characterized by frequency-dependent parameters and nonlinear simulators is the problem of mixed frequency/time since nonlinear components are represented only in the time domain, whereas when using sophisticated data analyzers or data-acquisition equipment, information about the system is often obtained as frequency-response samples instead of time-domain data. Commonly, the port response at discrete frequency points is available in the form of Y-parameters (admittance), Z-parameters (impedance), or S(scattering)-parameters obtained from broad-band measurements performed using commercial network analyzers or rigorous full-wave electromagnetic simulations. The frequency-dependent nature of the investigated problem generally requires: 1) the definition of the simulation model in frequency-domain; 2) the transformation from the frequency domain to the time domain; and 3) the transient analysis and time-domain solution [9]. For the continuous representation in the frequency domain, rational functions in the complex frequency $s = j\omega$ are used. From the rational functions, the inverse Fourier transforms for closed-form representation in the time domain are computed analytically. The resulting simulation process in the time domain is solved by using recursive convolution schemes for each port parameter. However, the numerical evaluation of convolution integrals can require high computational effort and can cause numerical stability problems. Alternatively, time-domain macromodel synthesis techniques can be applied to derive a corresponding equivalent circuit of the device-under-test (DUT). In that case, the CPU expenses for transient simulations are dependent on the order of the system model and the size of the synthesized lumped-element network. Furthermore, the major advantage in using macromodels is that a single model in a minimum realization can be obtained, as is desired in most cases [8].

Of particular interest is the development of reduced-order macromodels, which maintain the passivity of the original circuit, i.e., the resulting model entails that it should absorb active power for any set of applied voltages or currents at any frequency. Past experience has shown that simulations involving frequency-response data fitted by rational functions can sometimes lead to an unstable simulation, even though the discrete dataset for each port has been approximated using only stable poles. This requires proving passivity of the generated MIMO model for all frequencies from zero to infinity. Using transfer-function representations, a criterion for passivity is that $\mathrm{Re}\{\mathbf{Y}(s)\}$ or $\mathrm{Re}\{\mathbf{Z}(s)\}$ be positive definite (PD). Recently, several methods [1]–[4] have been proposed, which apply the PD criterion to ensure passivity. However, a difficulty generally encountered when using the PD criterion is that passivity can be guaranteed at discrete frequency points only. Thus, the eigenvalues of $\mathrm{Re}\{\mathbf{Y}(s)\}$ or $\mathrm{Re}\{\mathbf{Z}(s)\}$ have to be calculated at a large number of frequency points, within, but also beyond, the fitted frequency range in order to prove passivity for the gener-

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ated transfer-function model. In this paper, a different criterion to overcome this problem is presented. It enables to verify passivity of a developed model in the range of $0 \leq \omega < \infty$ by solving one Ricatti equation only. The criterion is suitable for passivity assessment of time-domain macromodels, which combines the advantages of minimal-order time-domain realization, and efficient assessment of model properties.

This paper is organized as follows. In Section II, different data-fitting techniques used to generate a continuous transfer-function representation from the discrete frequency response of multiports are discussed. Sections III and IV present a systematic approach to determining the corresponding minimal-order realization from the calculated transfer-function matrix and ensuring inherent system properties such as stability and passivity in a very efficient manner. How to generate the equivalent-circuit network is described in Section V. The models provide an accurate description of the investigated multiport systems within the specified frequency range and exhibit a considerably reduced complexity, which is verified by numerical examples in Section VI. Conclusions are drawn in Section VII.

$$\mathbf{Y}(\omega_i) = \begin{bmatrix} Y_{11}(\omega_i) & \ldots & Y_{1n}(\omega_i) \\ \vdots & \ddots & \vdots \\ Y_{n1}(\omega_i) & \ldots & Y_{nn}(\omega_i) \end{bmatrix}. \tag{1}$$

II. Data Fitting by Rational Functions

In practice, information about a system is often given in terms of frequency responses of the system at some discrete set of frequencies. Typically, the frequency-response data of the n-port to be modeled are provided in terms of Y-, Z-, or S-parameters at N discrete frequency points ω_i that cover the bandwidth of interest, e.g., as given in (1). The different frequency-dependent parameter matrices are related to each other by (2) and (3), where $\mathbf{Z}_0$ is a diagonal matrix containing the square roots of all port impedances as follows:

$$\mathbf{Y}(\omega_i) = \mathbf{Z}_0^{-1}\big(\mathbf{I} + \mathbf{S}(\omega_i)\big)^{-1}\big(\mathbf{I} - \mathbf{S}(\omega_i)\big)\mathbf{Z}_0^{-1} \tag{2}$$

$$\mathbf{Z}(\omega_i) = \mathbf{Z}_0\big(\mathbf{I} - \mathbf{S}(\omega_i)\big)^{-1}\big(\mathbf{I} + \mathbf{S}(\omega_i)\big)\mathbf{Z}_0. \tag{3}$$

The problem of fitting a real rational model to a given frequency response has been addressed by many authors [9]–[18]. In the traditional way, a one-port system is modeled as a fraction of two polynomials with real coefficients. The corresponding rational transfer-function matrix that approximates the n-port frequency parameter data can be written as

$$\mathbf{H}(s) = \frac{\mathbf{A}_0 + \mathbf{A}_1 s + \mathbf{A}_2 s^2 + \cdots + \mathbf{A}_\varepsilon s^\varepsilon}{1 + b_1 s + b_2 s^2 + \cdots + b_\eta s^\eta} \tag{4}$$

where b_0 is normalized to unity. The $\mathbf{A}_i$'s represent the $n \times n$ coefficient matrices of the numerator polynomials of the order ε and b_i's are the coefficient of the common denominator polynomial of the order η. The appropriate order is determined according to the chosen error criterion, e.g., a weighted least square type error criterion (5) as follows:

$$e_2 = \sum_{x=1}^{n}\sum_{y=1}^{n}\left(\frac{1}{2}\sum_{i=1}^{N} W_{xy,i}\left|H_{xy}(\omega_i)\sum_{k=0}^{\eta} b_k s_i^k - \sum_{k=0}^{\varepsilon} A_{xy,k} s_i^k\right|^2\right). \tag{5}$$

The transfer-function matrix $\mathbf{H}(s)$ can also be expressed in pole residue form

$$\mathbf{H}(s) = \mathbf{K}_0 + \sum_{i=1}^{\eta} \frac{\mathbf{K}_i}{s - p_i} \tag{6}$$

where the p_i's are the common pole and $\mathbf{K}_i$'s are residues of $\mathbf{H}(s)$. The coefficients of the transfer-function matrix (4) can be calculated using different complex curve-fitting techniques [9].

The model-based parameter-estimation method, e.g., represents the extension of Prony's approach to the treatment of frequency-domain data. This fitting technique was first applied by Miller *et al.* to the analysis of electromagnetic problems [13].

The $q = \eta + (\varepsilon + 1) \cdot n^2$ unknown coefficients in (4) are computed by applying a point-matching algorithm that forces the discrete data to be equal to $\mathbf{H}(\omega_i)$ at $q/2$ frequency points ω_i.

The linear system of equations for real and imaginary part of $\mathbf{H}(\omega_i)$ obtained from (7), shown at the bottom of this page, are straightforward to implement in computer code; however, for high-order approximations over a wide frequency range, the system is highly ill conditioned and nearly singular. This is because the ordinary power series $(1, \omega_i, \omega_i^2, \omega_i^3, \ldots)$ have a very

$$\begin{bmatrix} 1 & j\omega_0 & (j\omega_0)^2 & \ldots & (j\omega_0)^\varepsilon & -j\omega_0\mathbf{H}(\omega_0) & -(j\omega_0)^2\mathbf{H}(\omega_0) & \ldots & -(j\omega_0)^\eta\mathbf{H}(\omega_0) \\ 1 & j\omega_1 & (j\omega_1)^2 & \ldots & (j\omega_1)^\varepsilon & -j\omega_1\mathbf{H}(\omega_1) & -(j\omega_1)^2\mathbf{H}(\omega_1) & \ldots & -(j\omega_1)^\eta\mathbf{H}(\omega_1) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & j\omega_{q/2} & (j\omega_{q/2})^2 & \ldots & (j\omega_{q/2})^\varepsilon & -j\omega_{q/2}\mathbf{H}(\omega_{q/2}) & -(j\omega_{q/2})^2\mathbf{H}(\omega_{q/2}) & \ldots & -(j\omega_{q/2})^\eta\mathbf{H}(\omega_{q/2}) \end{bmatrix} \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_\varepsilon \\ b_1 \\ \vdots \\ b_\eta \end{bmatrix} = \begin{bmatrix} \mathbf{H}(\omega_0) \\ \mathbf{H}(\omega_1) \\ \vdots \\ \mathbf{H}(\omega_{q/2}) \end{bmatrix} \tag{7}$$

large dynamic range, and they become almost parallel at higher orders. The problem can be overcome by using normalized angular frequency values $\omega_i^* = \omega_i/\omega_0$ or splitting the frequency range in several sub-domains.

Another approach to circumvent the ill-conditioned system of equations is to replace the ordinary power series $(1, \omega_i, \omega_i^2, \omega_i^3, \ldots)$ in (6) by orthogonal polynomials, such as Chebyshev polynomials [14]. The Chebyshev polynomial of degree k is given by

$$T_k(x) = \cos(k \arccos x). \tag{8}$$

The equivalent power series for Chebyshev polynomials $(1, x, 2x^2 - 1, 4x^3 - 3x, \ldots)$ are orthogonal in the interval $[-1, 1]$ over a weight $(1 - x^2)^{-1/2}$. In addition, they have a small dynamic range that is bounded between -1 and 1 in the interval $[-1, 1]$. This makes the Chebyshev polynomials particularly well suited for approximations of very high order [14]. To this end, (7) needs to be rewritten using Chebyshev polynomials. After solving for the coefficients, the equivalent of the ordinary power series can be calculated efficiently using Clenshaw's recurrence formula.

A different method was developed by Gustavsen and Semlyen [15]. Their vector-fitting procedure determines the unknown residue values in (6) in an iterative manner starting with an estimated set of real or complex conjugated pairs of poles p_{0i}. For this purpose, the rational functions (9) and (10) are introduced as follows:

$$\sigma(s) = 1 + \sum_{i=1}^{\eta} \frac{\mathbf{K}_{xy,\sigma i}}{s - p_{0i}} \tag{9}$$

$$\mathbf{G}_{xy}(s) = \mathbf{K}_{xy,0} + \sum_{i=1}^{\eta} \frac{\mathbf{K}_{xy,i}}{s - p_{0i}} \tag{10}$$

$$\mathbf{G}_{xy}(j\omega_k) = \sigma_{xy}(j\omega_k)\mathbf{H}_{xy}(\omega_k). \tag{11}$$

The unknowns $\mathbf{K}_{xy,0}$, $\mathbf{K}_{xy,\sigma i}$, and $\mathbf{K}_{xy,i}$ are computed using sample values $\mathbf{H}_{xy}(\omega_k)$ and solving the linear system (11). Once the unknowns are computed, the transfer function for each port parameter in (6) can be expressed in the form

$$\mathbf{H}_{xy}(s) = \frac{\mathbf{G}_{xy}(s)}{\sigma_{xy}(s)}, \qquad x, y = 1, \ldots, n. \tag{12}$$

The error due to the approximation (12) is computed and the procedure is eventually reiterated by assuming as new starting poles the poles of $\mathbf{H}_{xy}(s)$ in (12). A critical aspect of the vector-fitting method is the choice of appropriate starting poles. Experimental analyses in [9] have demonstrated that, for smooth functions, i.e., without resonances, good results are obtained by using real starting poles, equally distributed along the frequency range of interest. The number of poles to use in the approximation in this case depends on the extension of the frequency interval in which the fitting model is applied. In the case of frequency-response data with multiple resonances, starting with complex conjugated poles should be considered. In order to avoid ill-conditioned problems in the system (11), the real parts of the initial poles are assumed to be 100 times smaller than the imaginary parts.

III. Minimal-Order Realization

Given a transfer-function matrix $\mathbf{H}(s)$, several forms of time-domain realizations can be obtained. The derivation of differential equations from a transfer-function system is referred to as macromodel synthesis. In general, a set of first-order differential equations in state–space domain can be described as

$$\begin{aligned} \frac{d}{dt}\mathbf{x}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \end{aligned} \tag{13}$$

where $\mathbf{A} \in \mathbb{R}^{m \times m}$, $\mathbf{B} \in \mathbb{R}^{m \times n}$, $\mathbf{C} \in \mathbb{R}^{n \times m}$, $\mathbf{D} \in \mathbb{R}^{n \times n}$, and m equals the number of states, i.e., the order of the system. Using, for example, the Y-matrix (1), the kth element of the input vector $\mathbf{u}(t)$ and output vector $\mathbf{y}(t)$ corresponds to the voltage $v_k(t)$ and current $i_k(t)$ at port k, respectively.

A state–space realization is said to be a minimal realization of a transfer function matrix $\mathbf{H}(s)$ if the system matrix $\mathbf{A}$ has the smallest possible dimension, i.e., the state–space system has the fewest number of states m. The smallest dimension is called the McMillan degree of $\mathbf{H}(s)$ [19], [20]. To calculate the minimal realization, first either the left or right co-prime factorization

$$\tilde{\mathbf{H}}(s) = \mathbf{H}(s) - \mathbf{H}(s = \infty) \tag{14}$$

$$\tilde{\mathbf{H}}(s) = \mathbf{D}_l(s)^{-1} \cdot \mathbf{N}_l(s) = \mathbf{N}_r(s) \cdot \mathbf{D}_r(s)^{-1} \tag{15}$$

is calculated. $\mathbf{D}_l$, $\mathbf{N}_l$, $\mathbf{D}_r$, and $\mathbf{N}_r \in \mathbf{P}(s)^{n \times n}$ are polynomial matrices in s. Considering the right co-prime factorization, $\mathbf{D}_r$ and $\mathbf{N}_r$ can be decomposed into a higher order coefficient matrix $\mathbf{D}_{hc} \in \mathbb{R}^{n \times n}$, where $\det(\mathbf{D}_{hc}) \neq 0$ and lower order coefficient matrices $\mathbf{D}_{lc}$ and $\mathbf{N}_{lc} \in \mathbb{R}^{n \times m}$, respectively,

$$\mathbf{D}_r(s) = \mathbf{D}_{hc}\mathbf{\Phi}(s) + \mathbf{D}_{lc}\mathbf{\Psi}(s) \tag{16}$$

$$\mathbf{N}_r(s) = \mathbf{N}_{lc}\mathbf{\Psi}(s) \tag{17}$$

with

$$\mathbf{\Phi}(s) = \operatorname*{diag}_{i=1,\ldots,n}\left(s^{k_i}\right) = \begin{bmatrix} s^{k_1} & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & s^{k_n} \end{bmatrix} \in \mathbb{C}^{n \times n} \tag{18}$$

$$\mathbf{\Psi}(s) = \operatorname*{blockdiag}_{i=1,\ldots,n}\left(\begin{bmatrix} 1 \\ s \\ \vdots \\ s^{k_n - 1} \end{bmatrix}\right) \in \mathbb{C}^{m \times n} \tag{19}$$

$$m = k_1 + k_2 + \cdots + k_n. \tag{20}$$

From (13)–(19), the state–space system is obtained by

$$\begin{aligned} -\mathbf{D}_{hc}^{-1}\mathbf{D}_{lc} &= \begin{bmatrix} a_{11} & \ldots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \ldots & a_{nm} \end{bmatrix} \\ \mathbf{D}_{hc}^{-1} &= \begin{bmatrix} b_{11} & \ldots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \ldots & b_{nn} \end{bmatrix} \end{aligned} \tag{21}$$

$$
\mathbf{A} = \begin{bmatrix} \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \ddots & \ddots & \ddots \\ a_{11} & \cdots & \cdots & \cdots & a_{1m} \end{bmatrix}_{k_1 x m} \\ \vdots \\ \begin{bmatrix} 0 & \ddots & \ddots & \ddots & \vdots \\ \vdots & & 0 & 1 & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ a_{n1} & \cdots & \cdots & \cdots & a_{nm} \end{bmatrix}_{k_n x m} \end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix} \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ b_{11} & \cdots & b_{1n} \end{bmatrix}_{k_1 x n} \\ \vdots \\ \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ b_{1n} & \cdots & b_{nn} \end{bmatrix}_{k_1 x n} \end{bmatrix} \tag{22}
$$

$$
\begin{aligned} \mathbf{C} =& \mathbf{N}_{lc} \\ \mathbf{D} =& \mathbf{H}(s = \infty). \end{aligned} \tag{23}
$$

In a similar manner, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ can be derived from the left co-prime factorization [21]. The minimal realization (22), (23) can easily be linked to standard nonlinear solvers or any general-purpose circuit simulator. Note that "minimal realization" implies minimal computational effort and reduced numerical stability problems.

IV. System Properties

Essential for the success of the proposed synthesis process is that the generated models meet the system behavior of the original circuits.

A. Stability

A critical aspect concerns the stability of the fitting model, which is assured if all roots p_i of the common denominator polynomial in (4) lie in the left-hand side of the complex plane, i.e., $\mathrm{Re}\{p_i\} \leq 0$. In general, stability can be enforced either as a constraint in the calculation of the rational approximation (4) or by applying correction techniques, such as reflection/contraction of unstable poles to the left half-plane, in a very simple manner.

B. Passivity

Ensuring passivity of the generated transfer-function representation is more difficult to handle. Passivity implies that the system cannot generate more energy than it absorbs, and that no passive termination of the system will cause the system to become unstable. A passive system is asymptotically stable. However, asymptotic stability, i.e., $\mathrm{Re}\{p_i\} < 0$, does not imply passivity. The loss of passivity can be a serious problem because transient simulations of the generated system model in a general circuit environment may encounter spurious oscillations

$$
\mathbf{I} - \mathbf{H}^T(j\omega) \cdot \mathbf{H}(-j\omega) \geq \mathbf{0} \tag{24}
$$

$$
\mathbf{H}^T(-j\omega) + \mathbf{H}(j\omega) \geq \mathbf{0}. \tag{25}
$$

Assuming the practical case of $\mathbf{H}(s)$ being symmetric and its coefficients $\mathbf{A}_i$ and b_i being only real valued, the network is passive if and only if, in case of S-matrix representation, the matrix (24) or, in case of Y- or Z-matrix representation, the matrix (25) is PD for $0 \leq \omega < \infty$ [3]–[5]. However, when testing the PD criterion for (24) or (25), respectively, one can encounter major difficulties. A closed-form solution for $0 \leq \omega < \infty$ is only available for one- or two-port systems. Alternatively, the numerical calculation of the eigenvalues of (24) and (25) for discrete frequency points over a very broad frequency range will cause high computational effort.

In this paper, a different criterion to overcome this problem is presented. Transforming the S-matrix to the corresponding Y- or Z-matrix representation using (2) and (3), the Kalman–Yakubovich–Popov (KYP) criterion [22] can be applied. The KYP criterion proves that a controllable and observable state–space system realization (13) of an admittance or impedance network is said to be passive for $0 \leq \omega < \infty$ if and only if there exist matrices $\mathbf{L}$, $\mathbf{W}$, and PD $\mathbf{P}$, satisfying the system of equations

$$
\begin{aligned} \mathbf{PA} + \mathbf{A}^T\mathbf{P} =& -\mathbf{L}^T\mathbf{L} \\ \mathbf{PB} =& \mathbf{C}^T - \mathbf{L}^T\mathbf{W} \\ \mathbf{W}^T\mathbf{W} =& \mathbf{D}^T + \mathbf{D}. \end{aligned} \tag{26}
$$

From (26), the Ricatti equation (27) can be derived as follows:

$$
\begin{aligned} &\mathbf{P}\left(\mathbf{I} + \mathbf{A}\right) + \left(\mathbf{I} + \mathbf{A}^T\right)\mathbf{P} + \cdots \\ &\quad \cdots + \left(\mathbf{C} - \mathbf{PB}\right)\left(\mathbf{D}^T + \mathbf{D}\right)^{-1}\left(\mathbf{C} - \mathbf{B}^T\mathbf{P}\right) = \mathbf{0}. \end{aligned} \tag{27}
$$

Thus, the proof that a given controllable and observable state–space system satisfies the passivity criterion reduces to the computation of the matrix $\mathbf{P}$ in (27) and its eigenvalues.

Since the minimal-order realization (22), (23) derived in Section III is controllable and observable, the above criterion can be applied to the proposed macromodel synthesis method. In addition, passivity of a developed model can be ensured by solving only a single Ricatti equation. Nonpassive models can be detected at an early stage of the modeling process with reduced computational effort.

Assuming that a rational approximation of (1) has been calculated with fairly high accuracy, but the corresponding minimal-order realization (22), (23) failed the modified KYP criterion (27), passivity can be enforced by making only a small correction to the rational approximation. The passivity enforcement based on linearization and constrained minimization is presented in detail in [4].

V. Equivalent Circuit

For those simulators that do not directly accept the differential equations as input, such as SPICE, the state–space system (22), (23) can be converted to an equivalent-circuit network consisting of passive elements and controlled voltage and current sources [2]. For the purpose of illustration, one considers a simple case of a two-port network with two states given by

$$
\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \\ \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \cdot \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}. \end{aligned} \tag{28}
$$

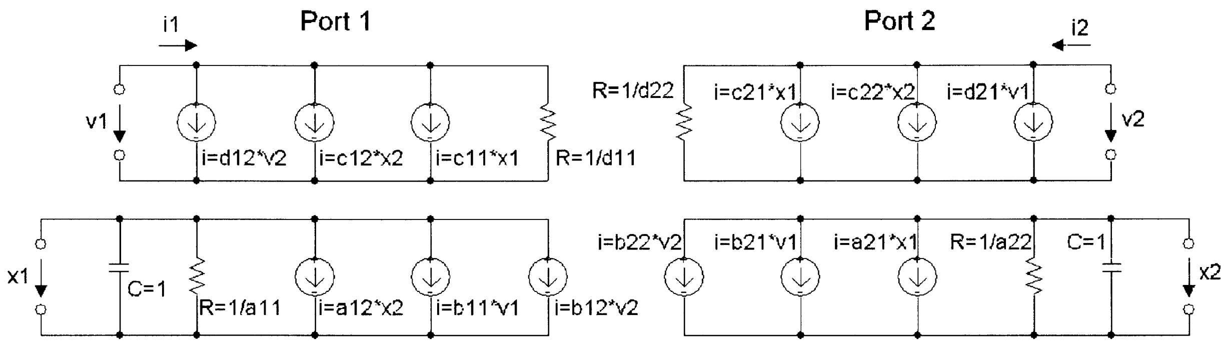

Fig. 1. Illustration of equivalent SPICE circuit generation from macromodel (28).

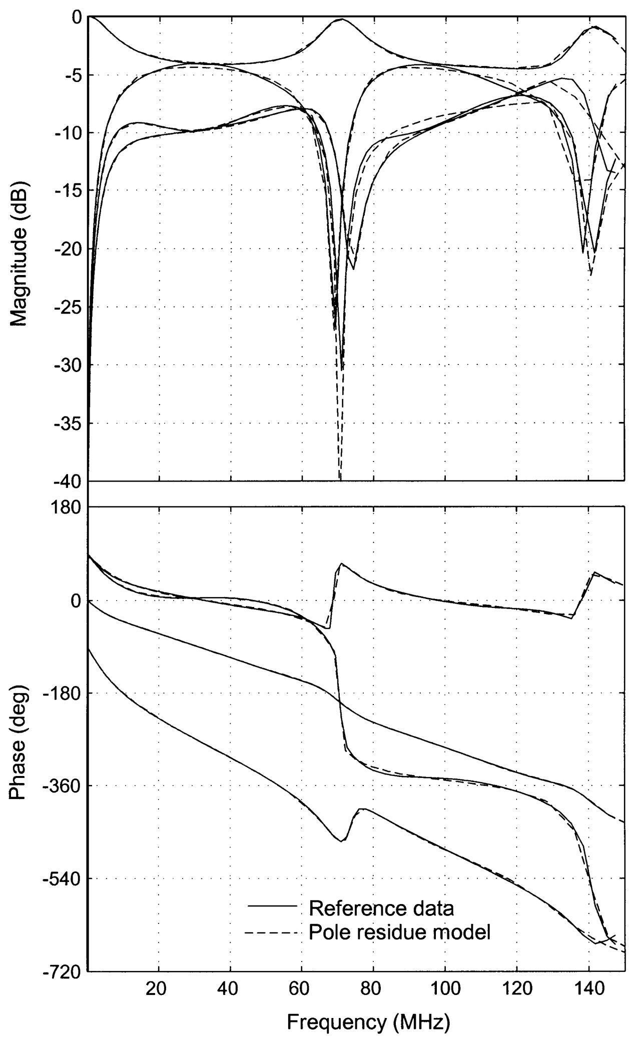

Fig. 2. Simulated S-parameters and calculated model of the investigated coupled flat cable over the frequency range from 50 kHz to 150 MHz.

The state–space system (28) can be constructed by controlled voltage and current sources, as shown in Fig. 1. The generalization of the above discussion in the case of a higher number of states or ports is straightforward.

VI. Experimental Results

In order to demonstrate the validity of this advanced approach, the following examples are considered.

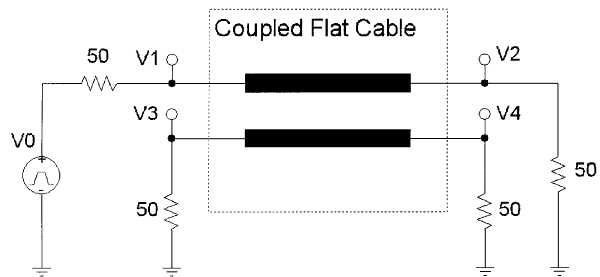

Fig. 3. Schematics of a coupled flat cable with input pulse at port 1 and terminations at ports 2–4.

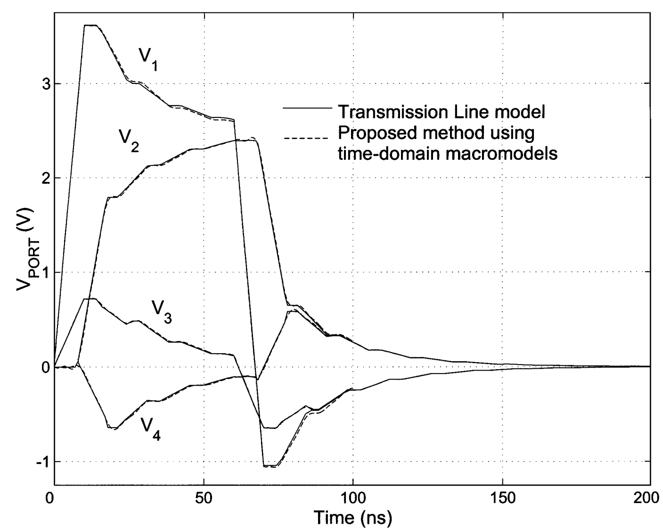

Fig. 4. Time response of coupled flat cable obtained for a 5-V/70-ns pulse with a rise/fall time of 10 ns.

A. Coupled Flat Cable

First, a coupled flat cable characterized by S-parameter data over the frequency range from 40 kHz to 150 MHz is modeled. The fitting algorithm achieves excellent agreement between the generated rational functions and the dataset used with a transfer-function matrix of twelfth order, as plotted in Fig. 2. Applying the above-described transformation, the corresponding state–space system of McMillan degree 12 is calculated. The extracted equivalent circuit model for the four-port is terminated with 50 Ω at ports 2–4. At port 1, a 50-Ω voltage source is exciting the system with a 5-V pulse of 70-ns duration (Fig. 3).

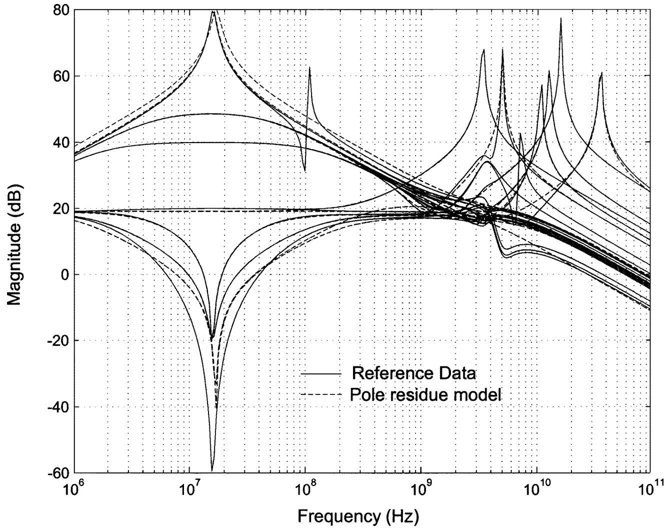

Fig. 5. Reference Z-parameters and fitting model of generic ten-port over the frequency range from 1 MHz to 100 GHz.

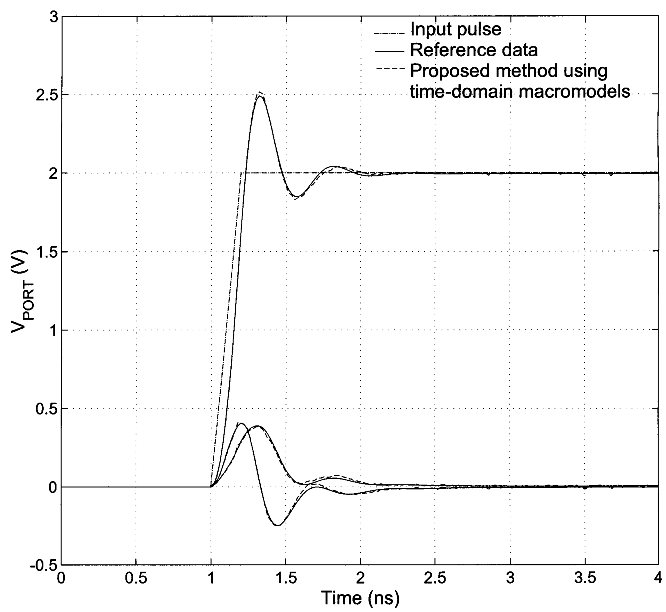

Fig. 6. Time response of generic ten-port obtained for a 2-V step with a rise time of 1.2 ns.

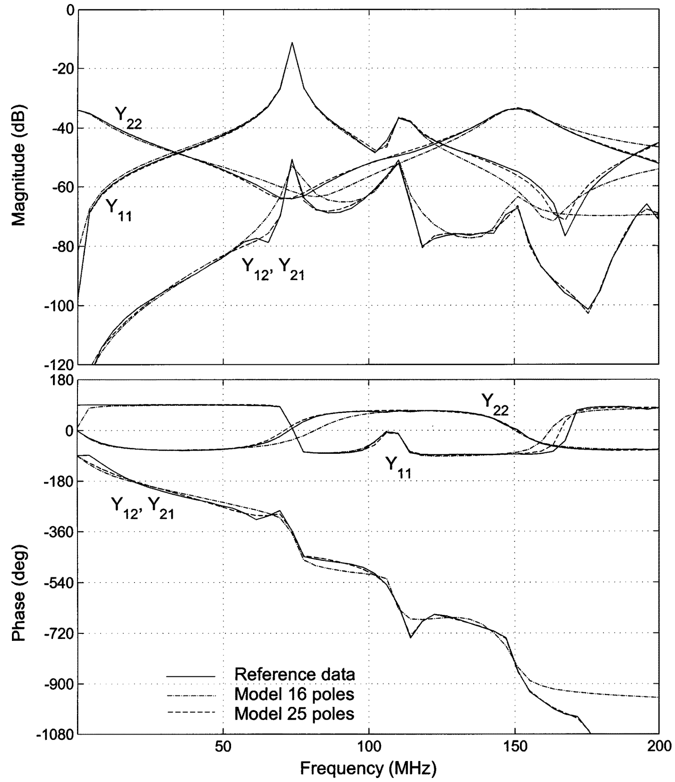

Fig. 7. Simulated Y-parameters and calculated models of sixteenth- and twenty-fifth order of the investigated harness antenna-coupling problem over the frequency range from 150 kHz to 200 MHz.

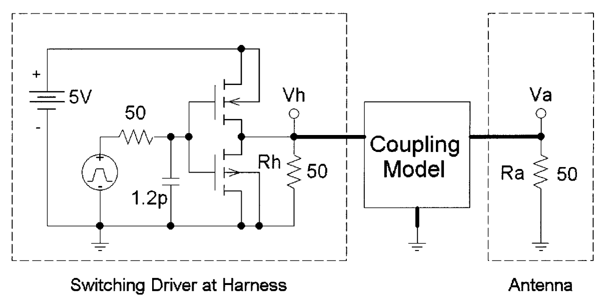

Fig. 8. Schematics of a harness antenna coupling with switching driver at the harness and 50 Ω at the antenna feeding point.

Unified transient simulations obtained using the calculated SPICE-compatible macromodel are compared to simulation results acquired with the original transmission-line model with frequency-dependent parameters in the frequency domain. As plotted in Fig. 4, excellent agreement is achieved.

B. Ten-Port System

As a second example, a generic lumped *RLC* network (ten-port) with resonance frequencies up to 40 GHz is considered. The reference Z-parameters of the ten-port and the fitted model are plotted in Fig. 5. Applying the above transformation process the corresponding time-domain macromodel is generated. The obtained network is excited at port 1; the remaining ports are terminated with 50 Ω. A comparison of the transient port responses of the reference system and generated time-domain macromodel at ports 2, 3, and 6 is shown in Fig. 6.

C. Cable Harness Antenna Coupling

In the next example, the coupling between a cable harness and an antenna is investigated. A full-wave field solver is used to calculate the S-parameters from 150 kHz to 200 MHz. From this, the Y-parameters are calculated and approximated by pole-residue models with 16 and 25 poles. The magnitude and phase responses of the data from simulation and pole-residue models are given in Fig. 7. Fig. 8. illustrates the termination of the synthesized lumped-element network. The transient response at the harness and antenna feeding point calculated with the proposed method compared with results obtained from mixed time- and

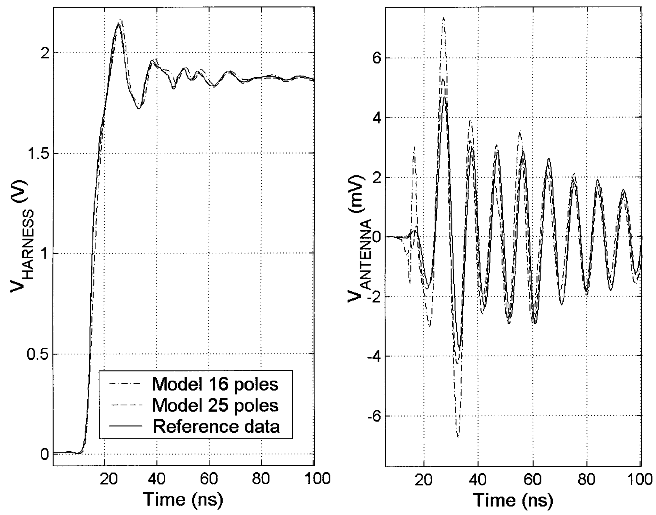

Fig. 9. Transient response at the harness (*left*) and antenna feeding point (*right*) obtained with the proposed method and mixed time-/frequency-domain simulations.

frequency-domain simulation shows excellent agreement, as depicted in Fig. 9.

VII. CONCLUSION

A systematic approach to extracting a system model of minimal order from components characterized by frequency-dependent data has been described. The proposed method enables transient simulations in a general circuit environment consisting of lumped/distributed elements and nonlinear devices, with increased numerical stability, decreased model complexity, and reduced computation times. These advantages, together with the ability to prove model properties such as stability and passivity in an efficient fashion, makes this method very suitable for system modeling and time-domain analysis of frequency-dependent data.

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